Palindromes

Competitive Programming

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Objectives

- Use DP to find all palindromic substrings.
- Learn the word “aibohphobia”

The Problem

Given a string s, find all the palindromic sub-strings.

- babba has three non-trivial palindromic substrings:
  - bb, abba, and bab

The algorithm

- Create a DP array $dp[|s|][|s|]$).
  - $dp[i][j]$ indicates if substring $s[i..j]$ is a palindrome.
  - Initialize diagonal to 1
- For each pair i, j, if $s[i] = s[j]$ then check if $s[i + 1..j - 1]$ is also a palindrome.
- Must iterate over smaller gap sizes first.

Code

```
1  int numPalindromes(string s) {
2    int i,j,gap,count;
3    vvb dp(s.length(),vb(s.length()),false);
4
5    count = 0;
6    for(i=0; i<s.length(); ++i)
7      dp[i][i] = true; // one character palindroms
8
9    // base casee: two character palindromes
10   for(i=1; i<s.length(); ++i)
11     if (s[i-1] == s[i]) {
12       dp[i-1][i] = true;
13       ++count;
14     }
```

Code, ctd

At this point we start from every 1 and "go up and right" to see if we can "grow" the palindrome.

```
15      // check odd palindromes
16      for(i=1; i<s.length()-1; ++i) {
17          int a=i-1, b=i+1;
18          while (a >= 0 && b < s.length() &&
19                  s[a] == s[b]) {
20              dp[a--][b++] = 1;
21              ++count;
22          }
23      }
24      // check even palindromes
25      for(i=1; i<s.length()-1; ++i) {
26          if (dp[i][i+1] == 0) continue;
27          int a=i-1, b=i+2;
28          while (a >= 0 && b < s.length() &&
29                  s[a] == s[b]) {
```

Example

- Example for babba

Matrix

	b	*a*	*b*	*b*	*a*
b					
a					
b					
b					
a					

Action

- Start with empty matrix

Example

- Example for `babba`

Matrix

	b	*a*	*b*	*b*	*a*
b	1				
a		1			
b			1		
b				1	
a					1

Action

- Start with empty matrix
- Initialize diagonal

Example

- Example for babba

Matrix

	b	*a*	*b*	*b*	*a*
b	1				
a		1			
b			1	1	
b				1	
a					1

Action

- Start with empty matrix
- Initialize diagonal
- Gap = 2, bb

Example

- Example for babba

Matrix

	b	*a*	*b*	*b*	*a*
b	1		1		
a		1			
b			1	1	
b				1	
a					1

Action

- Start with empty matrix
- Initialize diagonal
- Gap = 2, bb
- Gap = 3, bab

Example

- Example for babba

Matrix

	b	*a*	*b*	*b*	*a*
b	1		1		
a		1			1
b			1	1	
b				1	
a					1

Action

- Start with empty matrix
- Initialize diagonal
- Gap = 2, bb
- Gap = 3, bab
- Gap = 4, abba