

Rabin-Karp Algorithm

CS 491 – Competitive Programming

Dr. Mattox Beckman

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
DEPARTMENT OF COMPUTER SCIENCE

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Objectives

- ▶ Explain how a *rolling hash* works
- ▶ Use a rolling hash to find a pattern in a string quickly



Näive String Matching

A reminder of what not to do....

```
1  int find(string s, string desire) {
2      int found = -1;
3      for(int i=0; i<s.length() - desire.length(); ++i) {
4          found = 0;
5          for(int j=0; j<desire.length(); ++j)
6              if (s[i] != s[j])
7                  break;
8              else ++found;
9          if (found == desire.length())
10             return i;
11     } // end for i
12     return -1; // not found
13 }
```

► Time complexity is $\mathcal{O}(|P||T|)$

Hash Functions

- ▶ Remember hash functions!
 - ▶ $h(s)$ should be *fast to compute*
 - ▶ $h(s_1) = h(s_2)$ *probably* means $s_1 = s_2$
 - ▶ $h(s_1) \neq h(s_2)$ *definitely* means $s_1 \neq s_2$
- ▶ Can this help us with string matching?

Hashing

Consider this code

```
14 int find(string s, string desire) {
15     int found = -1;
16     for(int i=0; i<s.length() - desire.length(); ++i) {
17         if (h(desire) == h(s.substr(i,desire.length()))) &&
18             desire = s.substr(i,desire.length()))
19             return i
20     } // end for i
21     return -1; // not found
22 }
```

► How about now?

Rolling Hashes

- ▶ Consider this hash function:

$$h(c_0 \cdots c_{n-1}) = c_0 a^n + c_1 a^{n-1} + c_2 a^{n-2} + \cdots + c_{n-1} \text{ modulo } b$$

- ▶ a is a constant (256 is reasonable)
 - ▶ b is a large prime number (let's use 100007)
 - ▶ c_i is the i th character in a string.
- ▶ Try it yourself!
 - ▶ Compute the hash for abc
 - ▶ Compute the hash for bci
 - ▶ Hint: ASCII for a is 95, i is 103

Rolling Hashes, ctd

- ▶ Consider this hash function:

$$h(c_0 \cdots c_{n-1}) = c_0 a^n + c_1 a^{n-1} + c_2 a^{n-2} + \cdots + c_{n-1} \text{ modulo } b$$

- ▶ $h(\text{"abc"}) = 95 \times 256^2 + 96 \times 256 + 97 \text{ mod } 100007 = 50159$
- ▶ $h(\text{"bci"}) = 96 \times 256^2 + 97 \times 256 + 103 \text{ mod } 100007 = 15950$
- ▶ Can you convert from one to the other quickly?
 - ▶ Add 100007 to prevent "going negative"
 - ▶ "Subtract off" a by subtracting $(95 \times 256^2 \text{ mod } 100007)$
 - ▶ Multiply the remainder by 256 and modulo 100007.
 - ▶ Add 103
 - ▶ Take the modulus again.
- ▶ So: $h(\text{"bci"}) = ((h(\text{"abc"}) + 100007 + (95 \times 256^2 \text{ mod } 100007)) \times 256 + 103) \text{ mod } 100007$

Setting up

```
23  int a = 256;
24  int b = 100007;
25
26  int pow = 1;
27  for(i=0; i<desire.length(); ++i)
28      pow = (pow * a) % b;
29
30  int hash_s = 0;
31  int hash_d = 0;
32
33  for(i=0; i<desire.length(); ++i) { % assume desire < s
34      hash_s = hash_s * a + s[i];
35      hash_d = hash_d * a + desire[i];
36  }
```


Matching Part

```
37 while (i < s.length() - desire.length()) {
38     if (hash_s == hash_d &&
39         desire = s.substr(i,desire.length()))
40         return i;
41     // Subtract out first letter
42     hash_s = ((b + hash_s) - (s[i] * pow % b) * a % b);
43     // Add new letter to end
44     hash_s += s[i+desire.length()];
45     // Always keep modding!
46     hash_s = hash_s % b;
47 }
48 return -1; // if failed.
```